thank G. L. Burnett, who made the measurements, and W. P. Clark for his helpful suggestions.

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Wide-Band Phase Locking and Phase Shifting Using Feedback Control of Oscillators

DAVID RUBIN

***Abstract*—Two YIG-tuned Gunn oscillators[1] have been frequency locked over a 3-GHz range in *X*-band. Injected power into the locked oscillator was approximately 10 dB below its output power. Analog frequency-tracking circuitry was used together with phase-comparator feedback to achieve output phases which remained within ±10° over a 1.2 GHz bandwidth. Controlled phase shifting was obtained by applying dc voltages within the feedback loop.**

In order for oscillators to phase track one another over large frequency ranges, it is first necessary that their free-running frequencies be brought to within the appropriate range [1]

$$\Delta f \leq \frac{f_o}{2Q_o} \sqrt{P_i/P_o} \tag{1}$$

where $\Delta f = |f_i - f_o|$ is the frequency difference between the master and slave oscillators without signal injection, P_i is the power injected into the slave oscillator cavity, P_o is the slave oscillator output power, and Q_o is the loaded Q of the slave cavity. The above formula holds for $P_i \ll P_o$. Even with oscillators and magnetic structures built to extremely close mechanical tolerances, with the same current through the two coils their free-running frequencies may differ as much as 200 MHz and have linearity variations of ±15 MHz throughout the X-band region. Frequency-locking measurements at 10 GHz, wherein the locking range of the slave oscillator is measured as a function of the P_i/P_o ratio, gave a Q_o of 450, indicating that the slave frequency must be held to within 3.5 MHz of the master frequency for locking to occur with $P_i/P_o = 0.1$.

In addition to the main frequency-controlling current, which was series connected to both YIG coils,[1] a small additional current was added to one of the coils. Without injection locking, the latter current was varied until the two oscillator frequencies were approximately the same, as noted on a spectrum analyzer. This additional current was noted at incremental frequencies throughout X band, first with frequency increasing, then with frequency decreasing. The currents were different for the two cases due to hysteresis of the magnetic circuit. Fig. 1 shows the additional currents needed for frequency tracking, and also shows a current synthesized by the circuit of Fig. 2.

For the frequency-tracking circuit, one connection serves as both input and output. The input voltage is taken from the slave-oscillator YIG coil and rises linearly with frequency. Operational amplifiers 1 and 2 serve as voltage follower and inverter, respectively, and do not load the YIG coils. At frequencies below 7.5 GHz, all diodes are held below breakdown, and a constant current enters operational amplifier number 3 through a resistor connected to $-V_s$. As the frequency is raised, each diode in turn breaks down, first removing, then adding current to the input of operational amplifier number 3. The input voltage at which a diode breaks down and the amount of current conducted through a diode, is dependent on the two resistors connected to it. The current entering operational amplifier number 3, and hence its output voltage, is thereby made nonlinear with respect to the input frequency.

Operational amplifier number 4 is called a Howland pump [2]. It delivers an output current proportional to the voltage input, i.e., even though the load itself is a resistance in series with a voltage source. The delivered current is shown as the solid line in Fig. 1. With this circuit the YIG oscillators could track to 2 GHz. Also note that a similar method could be used for tracking varactor-tuned oscillators, wherein the input would be a voltage proportional to frequency, and the nonlinear output voltage would go to a varactor diode instead of being returned to the input. In the latter case, the Howland current pump would be unnecessary.

Fig. 3 shows a block diagram including the feedback method used. The phase comparator used is a 3-dB quadrature coupler with input and normally isolated terminals connected to the input RF sources and output terminals connected to diode detectors. The detected voltage difference is proportional to the sine of the input phase difference. This provides the input for a dc amplifier and pump circuit. This in turn feeds current to the slave-oscillator YIG coil, which tends to adjust its free-running frequency until the inputs to the phase comparator are in phase. If a dc reference voltage E_Φ is connected to one of the terminals of the latter differential amplifier, the feedback circuitry will adjust the current until the output voltage of the phase comparator provides approximately the same voltage to the other terminal of the difference amplifier.

Hence one could adjust the gain of the pump circuit so that if 1 V at reference terminal E_Φ represented +90° phase shift, one could set a reference voltage E_Φ' so that $\sin \Phi = E_\Phi'$ and get a phase range that in practice will be less than ±90°. For accuracy over wide bandwidths, power levelers should be used so that P_i/P_o is constant. For the experiments to be described the power output of both YIG oscillators varied similarly over the band so that $P_i/P_o = 0.1$ over most of the frequency range.

With feedback the locking range at any given frequency is increased [3]. Likewise a given phase change due to the differences in the free-running frequencies is reduced. This can be seen from the following argument.

Let $\Delta\phi$ and $\Delta\phi'$ be the phase differences between YIG oscillator outputs with and without feedback as seen at the input terminals of the phase comparator. Corresponding to $\Delta\phi$ and $\Delta\phi'$ are the free-running slave-oscillator frequencies f_o and f_o'. These frequencies are linearly dependent on the YIG coil current, so that $f_o = c_1 I_o + c_2$ and $f_o' = c_1(I_o + I_f) + c_2$, where I_f is the current fed back from the phase comparator circuit. The latter is proportional to the sine of the phase difference, i.e., $I_f = c_3 \sin \Delta\phi' \approx c_3 \Delta\phi'$ for small phase differences. The Adler formula [1] becomes, for small angles,

$$\Delta\phi \approx c_4(f_o - f_i) \tag{2}$$

$$\Delta\phi' \approx c_4(f_o' - f_i) \tag{3}$$

where

$$c_4 \approx \frac{2Q_o}{f_i} \sqrt{\frac{P_o}{P_i}} \tag{4}$$

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The author is with the Microwave Technical Division, Naval Electronics Laboratory Center, San Diego, Calif. 92152.
[1] Watkins-Johnson Model 5008-2.

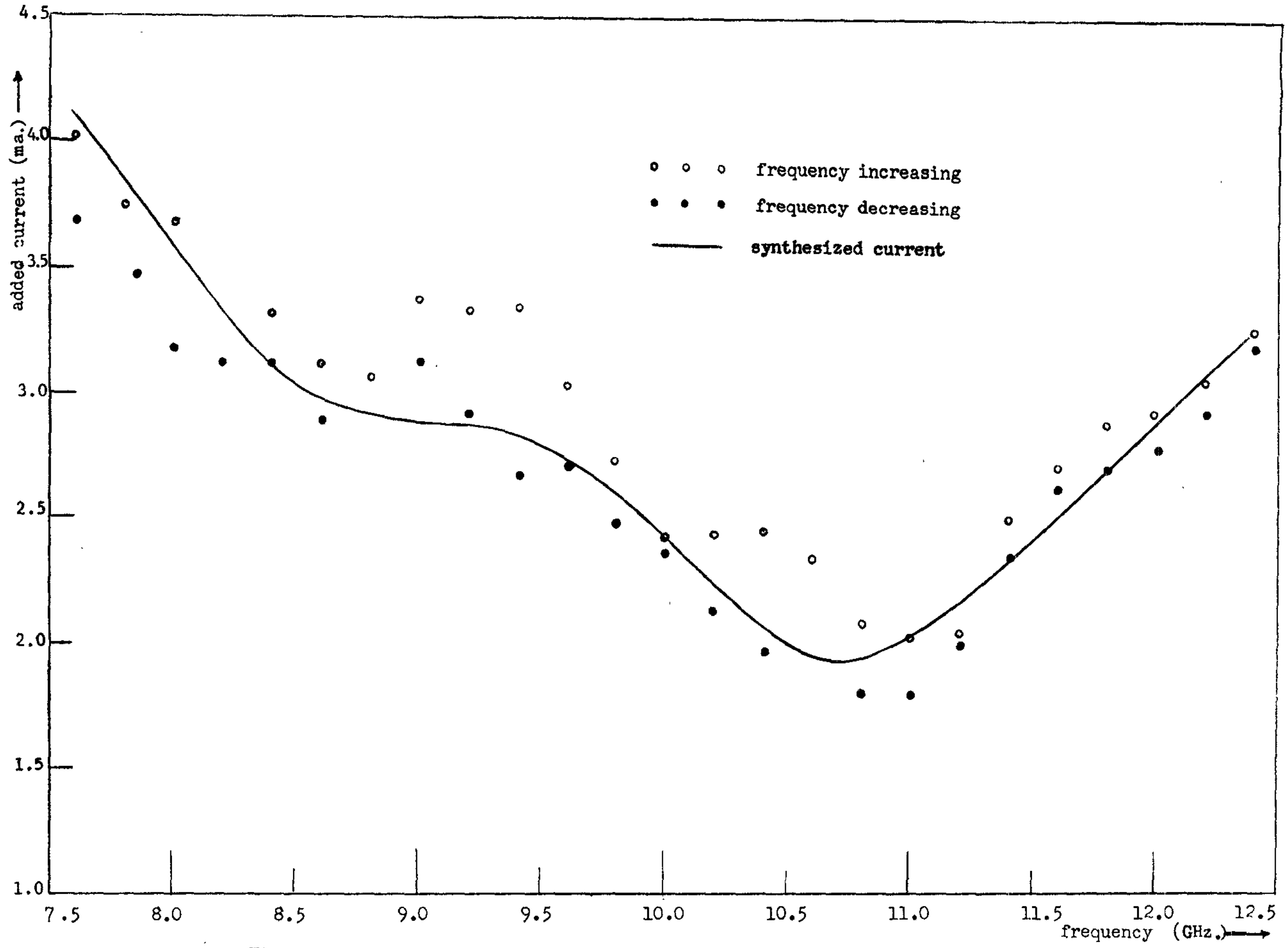

Fig. 1. Current excess required by slave YIG coil in order to frequency track master oscillator.

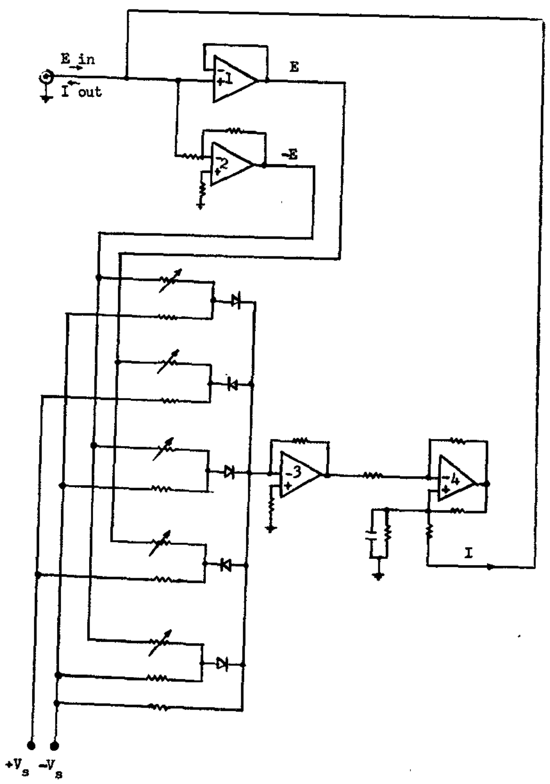

Fig. 2. Current-synthesis circuit used for frequency tracking YIG-tuned oscillators.

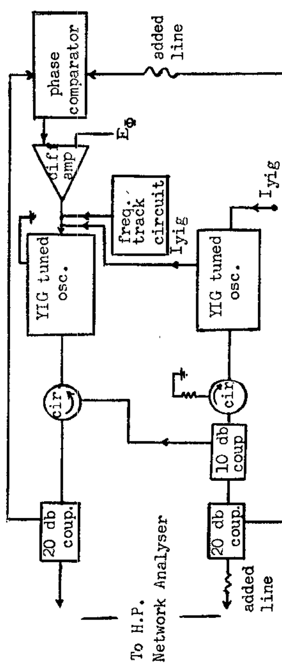

Fig. 3. Overall diagram showing phased-locked loop and method of phase shifting.

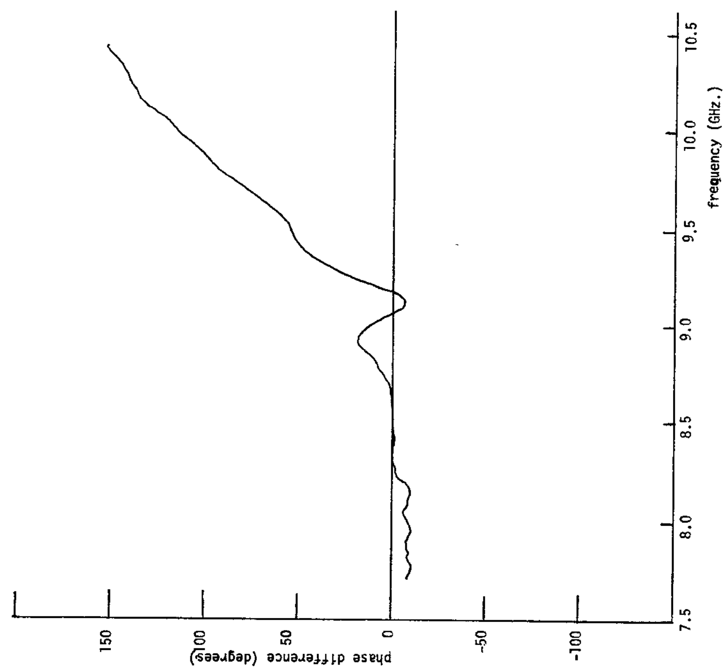

Fig. 4. Output phase difference versus frequency for the two YIG oscillators. Output line intentionally offset.

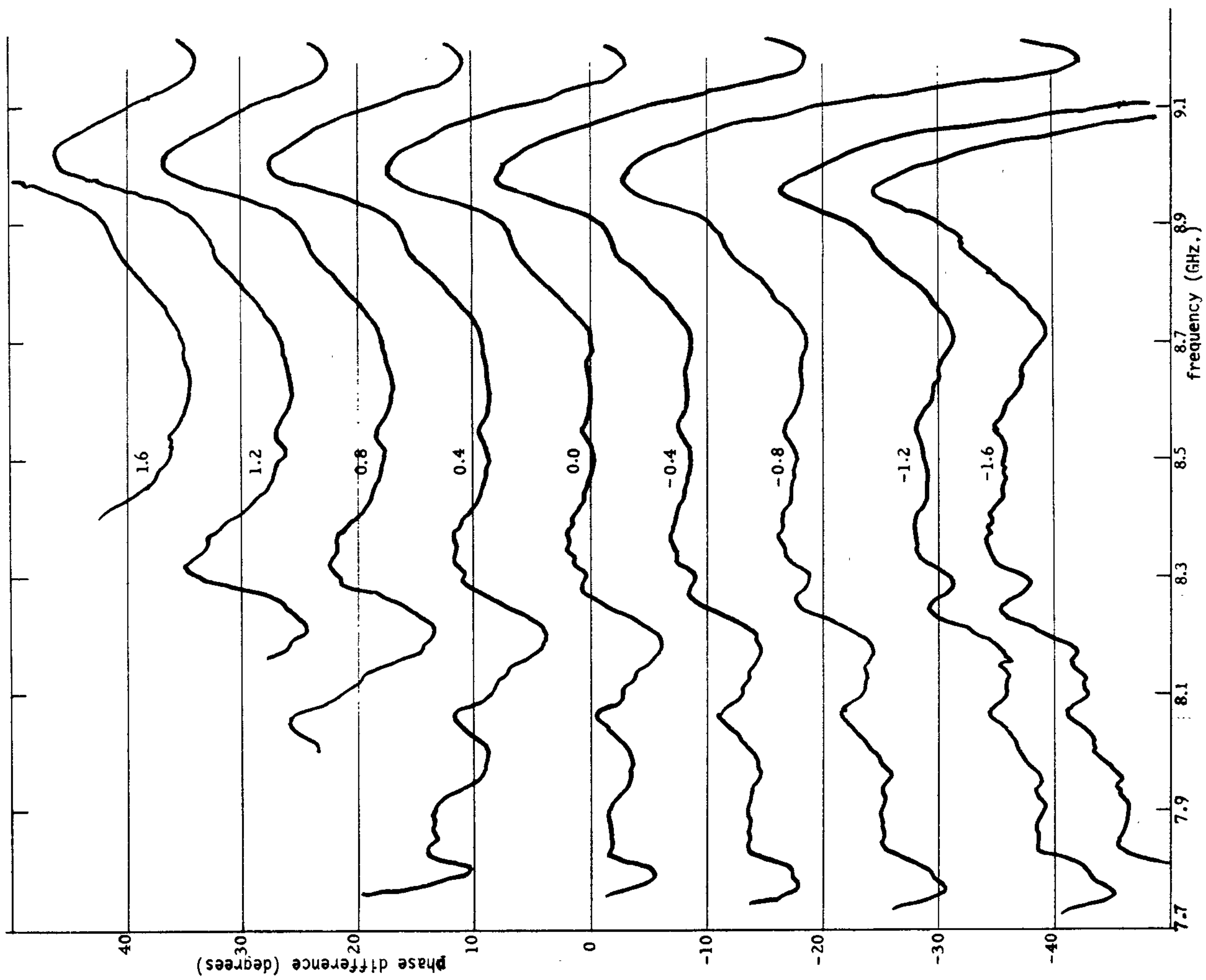

Fig. 5. Output phase difference versus frequency as a function of reference voltage E_Φ.

hence,

$$I \approx c_3c_4(f_o' - f_i)$$

and

$$f_o' \approx f_o + c_1c_3c_4(f_o' - f_o).$$

Solving for f_o' we have from (2) and (3)

$$\frac{\Delta\phi'}{\Delta\phi} \approx \frac{f_o' - f_i}{f_o - f_i} \approx \frac{1}{1 - c_1c_3c_4}. \tag{5}$$

For the YIG oscillators used $C_1 = 17$ MHz/mA. For the feedback system used $C_3 = -(360)/(2\pi) \times 8 \times 10^{-3}$ mA/rad.

Inserting values previously given into (4) we find $C_4 = 0.3$ rad/MHz. Hence,

$$\frac{\Delta\phi'}{\Delta\phi} \approx \frac{1}{3}.$$

This reduction in phase shift with feedback was observed at fixed frequencies.

It is extremely important to adjust the lengths of lines to the phase comparator so that, over the frequency range to be used, there will be near zero phase difference at the terminals at the center of the locking range. This is best done with a network analyzer taking the place of the phase comparator. By manually adding a small current to the slave-oscillator coil, one can find a particular phase difference θ_1 at the center of the locking range at some frequency f_1. The phase difference θ_2 is then found at the center of the locking range at another frequency f_2. For a path length difference L between the locked and unlocked sources,

$$\theta_1 = \frac{360}{c} f_1 L$$

$$\theta_2 = \frac{360}{c} f_2 L$$

where c represents velocity of propagation of the lines used, and θ_1 and θ_2 are in degrees. Then,

$$L = \frac{c}{360} \frac{\theta_2 - \theta_1}{f_2 - f_1}.$$

With the correct cable length, the output of the comparator should always change polarity in the same direction when the slave-oscillator free-running frequency becomes higher than the master-oscillator frequency. A similar method is also used to equalize the lengths of signal paths to the final output.

Fig. 4 shows the degree of locking obtained using the above methods. The signal paths to the network analyzer were made slightly unequal in order to decrease the change in phase angle at the low end of the band. For this reason the phase difference at the output is greater than $+90°$ over part of the phase-locked frequency range.

Under the above conditions, the voltage E_Φ was set to incremental values, and a slowly changing ramp current was applied to the series YIG coils. A plot of output phase with frequency (Fig. 5) is shown as a function of the applied voltage E_Φ. As expected, the output phase differences varied linearly with the applied voltage for small phase angles.

It appears that the degree of success, using this method of phase shifting, is very much dependent on the accuracy of frequency tracking. It is expected that varacter-tuned oscillators, in which hysteresis has no effect, would be preferable to the YIG oscillators used in this experiment.

Finally, the feedback method should also correct for differences in phase tracking between similar amplifiers used before the final couplers.

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Notes on the Conjugate Matched Two-Port as an UHF Amplifier

C. S. GLEDHILL AND M. F. ABULELA

***Abstract*—Because of the relative simplicity of measurement of scattering parameters of active two-ports at frequencies up to the lower microwave region, investigations have been made into the application of these parameters to the design of UHF amplifiers. The theory of generalized scattering parameters has been developed by Kurokawa [1], applied to two-port power-flow analysis by Bodway [2], and used in the design of a single-stage UHF amplifier by Froehner [3]. In this last paper, the bandwidth limitations imposed by the matching networks were not considered, nor was the capacitive matching arrangement to a purely resistive load fully developed. Both of these topics are the subject of this short paper, in which the relevant design expressions are also given.**

Discussion

In this short paper it will be assumed that the elements S_{ij} of the n-port scattering matrix are well understood and can be used without further definition.

It is shown in the Appendix that for a one-port, for $S_{11} = \Gamma$, its Q can be written

$$Q = \frac{|\Gamma - \Gamma^*|}{1 - |\Gamma|^2} \tag{1}$$

and further, it has been shown [2] that an unconditionally stable two-port can be simultaneously matched at both ports, and under these conditions yields its maximum transducer gain. The source and load reflection coefficients to satisfy this condition were found to be, respectively,

$$\Gamma_{ms} = C_1^* \left\{ \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2|C_1|} \right\} \tag{2}$$

and

$$\Gamma_{ml} = C_2^* \left\{ \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2|C_2|} \right\} \tag{3}$$

in which

$$\begin{aligned} C_1 &= S_{11} - S_{22}^*\Delta \\ C_2 &= S_{22} - S_{11}^*\Delta \\ B_1 &= 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \\ B_2 &= 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \\ \Delta &= S_{11}S_{22} - S_{12}S_{21}. \end{aligned} \tag{4}$$

For a narrow-band amplifier, the Q is related to the center frequency ω_0, and the bandwidth $\Delta\omega$ by the well-known relation

$$Q = \frac{\omega_0}{\omega\Delta} \tag{5}$$

and obviously, from (2)–(4), Γ_{ms} and Γ_{ml} are calculable from the measured scattering parameters.

Hence, from these and (1), the maximum bandwidth for this type of load-matching condition is:

$$\Delta\omega = \frac{\omega_0(1 - |\Gamma_{ml}|^2)}{|\Gamma_{ml} - \Gamma_{ml}^*|}. \tag{6}$$

This simple relationship is an obvious aid to design, as it determines the maximum bandwidth available from a given device at a given center frequency (for an unconditionally stable maximum transducer gain).

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The authors are with the Department of Electrical Engineering and Electronics, University of Manchester Institute of Science and Technology, Manchester, M60 1QD, England.